[This question paper contains 2 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 12910 **K**

Unique Paper Code : 2354001001

Name of the Paper : GE Fundamentals of Calculus

Name of the Course : **Common Prog. Group**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory and carry equal marks.
3. This question paper has **six** questions.
4. Attempt any **two** parts from each question.

1. a) (i) Show that $lim_{x\to 0}\frac{x-|x|}{x}$ does not exist.

   (ii) Evaluate $lim_{x\to 1}\frac{x-1}{\sqrt{x}-1}$

   b) Examine the continuity at $x = 1$ and $x = 2$ of the function $f(x)$ defined below

$$f(x) = \begin{cases} 2x, & 0 \le x \le 1 \\ 2-x, & 1 < x \le 2 \\ x^2 - 2x, & x > 2 \end{cases}$$

   Also, the type of discontinuity, if any.

   c) Discuss the differentiability at $x = 0$ of the function $f(x)$ defined below

$$f(x) = x\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}},\ x \neq 0 \text{ and } f(0) = 0.$$

2. a) Find the n^{th} differential coefficient of $tan^{-1}\{2x/(1-x^2)\}$.

   b) If $y = \cos(m\,sin^{-1}x)$, evaluate $y_n(0)$.

   c) If $u = sin^{-1}\left\{\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}\right\}$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{144}\tan u\,(13 + tan^2 u).$$

3. a) State Rolle's theorem and verify the same for the function $f(x) = (x-a)^2(x-b)^3$ for all $x \in [a, b]$.

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b) State and prove Lagrange's mean value theorem.

c) Verify the Cauchy's mean value theorem for the following pair of functions:
(i) $f(x)=\sin x$ and $g(x)=\cos x$ in the domain $[-\pi/2,0]$.
(ii) $f(x)=\sqrt{x}$ and $g(x)=\frac{1}{\sqrt{x}}$ in the domain $[1,5]$.

4. a) State Taylor's theorem and prove that

$$sinax = ax-\frac{(ax)^3}{3!}+\frac{(ax)^5}{5!}- \ldots\ldots+\frac{(ax)^{n-1}}{(n-1)!}\sin\left(\frac{n-1}{2}\pi\right)+\frac{(ax)^n}{n!}\sin\left(a\theta x+\frac{n\pi}{2}\right),$$

where a is a nonzero real number and $0<\theta<1$.

b) Find the Maclaurin's series of $\log(1+x)\ \forall -1<x\leq 1$, with Cauchy's form of the remainder.

c) Evaluate the following:
(i) $lim_{x\to 0}\left(\frac{1}{x^2}-\cot^2 x\right)$ (ii) $lim_{x\to 0}\frac{x-\sin x}{x^3}$

5. a) Trace the curve

$$r = 2 + 3cos\theta, \qquad 0\leq\theta\leq 2\pi$$

b) Sketch the graph of the function

$$y = x^3 - 3x + 2$$

c) Find the oblique asymptotes of the curve

$$y^3 - x^2y - 2xy^2 + 2x^3 - 7xy + 3y^2 + 2x^2 + 2x + 2y + 1 = 0$$

6. a) Determine the intervals of concavity and points of inflexion of the function

$$f(x) = x^3 - 3x^2 + 1.$$

b) Use the second derivative test to find the points of local maxima and minima of the function $f(x) = 12x^5 - 45x^4 + 40x^3 + 6$. Also determine the intervals on which f(x) is increasing and decreasing.

c) Determine the critical points of the function $f(x) = \frac{x+1}{x^2+3}$. Also, find the horizontal and vertical asymptotes.